Odd Time Transitions

How one meter slips into another — the rails, the map, and the Long / Short catalogue that feeds them

by Dr Zlatko Baracskai • www.zlatko.hu

The question — what makes a transition possible

A meter is easy to state and hard to leave. This sheet is not a catalogue of odd times for their own sake; it is about the *moves between them* — which pairs of bar-lengths can be joined by a single change, which need a detour, and which turn out to be further apart than their arithmetic suggests. Everything below exists to make that question answerable: a vocabulary of Long/Short cells, three tables that each read the vocabulary as a kind of transition, and a lattice that turns "how do I get from 7 to 8?" into a shortest path of a definite length.

The move itself is always the same gesture. **Hold the shape, change the cell sizes.** Play `LLS` as 2+2+1 and you are in 5; play the same `LLS` as 3+3+2 and you are in 8 — identical body movement, identical accent contour, nothing altered but the size of the two durations. That is a transition, and what follows is an account of which ones exist.

Derivation — where the transitionable rhythms come from

- **Two building blocks.** Every rhythm here uses only two note-lengths: a **Short** (`S`) and a **Long** (`L`), with Long > Short. A *pattern* is just the order you play them in — a shape in time — before you decide exactly how big Short and Long are. The shape is what survives a transition; the sizes are what change.
- **Adding, not dividing.** Instead of slicing a bar into equal parts, we *add* cells end to end. The total is Number = $a \cdot$ Short + $b \cdot$ Long (a Shorts plus b Longs). This is exactly how the familiar odd metres are built — 5 = 2+3, 7 = 2+2+3, 8 = 3+3+2 — and it is why a bar-length can be *moved* at all: nudge a duration and the total follows.
- **Listing every shape.** For each **length** n (how many notes) write down every order of Short/Long that has *at least one of each*. Throw away the two "flat" rows (all Shorts, all Longs), since those are just a plain even pulse — and a flat row has no contrast to hold constant, so it can carry no transition. A shortcut for counting: write Short=0, Long=1 and you are simply listing binary numbers, which gives $2^n - 2$ shapes per length — 2+6+14+30 = **52** shapes in all for lengths 2 to 5.
- **Capping the contrast at 3:1.** We allow a Long to be at most three Shorts (Long $\leq 3 \cdot$ Short). Anything more lopsided is discarded: a 4:1 cell such as 4+1, or a 5:1 such as 5+1, no longer reads as "Long against Short" — the Short is so brief it collapses into a flam or a rest, and the ear hears one weighted pulse instead of two. The cap keeps every cell inside a singable, clearly felt contrast — and, as Part 4 shows, it is also what punches the holes in the map.
- **Keeping only primitives.** We also drop any rhythm that is merely a scaled-up copy of a smaller one. 2+4 is just 1+2 played twice as slow; 4+2 is 2+1 doubled; 3+6 is 1+2 tripled. They carry the *same* shape and feel, so a move onto one of them is no transition at all — it is a tempo change. We keep only the smallest representative — the *primitive*, where Short and Long share no common factor — and throw the multiples away. The price of that decision is paid in full on the last page.
- **Dropping one of each mirror pair.** A pattern and its *retrograde* — the same cells played backwards (`LSS`→`SSL`) — have the identical cycle and the identical transition behaviour; one is just the other read end-first. Since this whole sheet is about *where a rhythm can travel*, a retrograde follows exactly the same routes and tells us nothing new, so we keep only one of each pair. A *palindrome* (`SLS`, `LSL`, `LSSL`) is already its own retrograde and simply stays.

- **What the three filters leave.** Across lengths 2 to 4 the 3:1 cap, the primitive rule and the no-retrograde rule leave exactly **37 rhythms** — the rows of Table 1, and the complete stock of departure and arrival points for every move in this sheet. They are spread unevenly. A four-cell bar of 7 admits only one value pair, Short=1 with Long=2, so its only four-cell rhythms are the two non-mirror orderings of 2+2+2+1 (`LLLS`, `LLSL`). A three-cell bar of 6, by contrast, has no valid pair at all and so does not occur — a bar you simply cannot arrive at that way. Meanwhile one shape can be realized at several lengths, so the *same* gesture surfaces at 5, 8, 11, ... — the shared relatives that let one rhythm be heard *against* another as a polyrhythm, and that let a player walk from one meter to the next. Why some lengths are crowded and others empty is the geometry taken up on the last page.

Three tables, three kinds of transition

- **Table 1 — the vocabulary, by cycle.** The plain catalogue, sorted by total bar length N $(3, 4, 5, 7, 8, 9)$. It answers "what can I fit into a bar of N?" — which is to say, what there is to leave *from* and to arrive *at*. The *Ratio* column $(L : S)$ tells you how uneven the two lengths are.
- **Table 2 — transition along a rail.** Fix one shape and climb the pulse at the gentlest contrast ($l = s+1$): `LS` = 3+2, 4+3, 5+4, ...; `LLS` = 2+2+1, 3+3+2, 4+4+3, ... Each row is the same gesture at a wider pulse, so a block is a *trajectory to ever-higher cycle numbers* — one per shape — with the step size set by the rhythm's *density* (its cell-count n). These rows are the elementary moves; every route on the last page is built out of them.
- **Table 3 — transition across the ratios.** The *same* rhythms regrouped so all versions of one shape sit together, sorted from the gentlest ratio to the steepest. Reading down a block, each move to the neighbouring ratio is a *natural jump between cycle numbers* — the most musical way to slip from one meter into another, and the move that gets you off one rail and onto another.

Table 1 — the vocabulary, ordered by cycle

Every departure and arrival point in this sheet. Mixed L/S patterns in 3–9 pulses ($N = a\,s + b\,l$, $s < l$, ratio $L{:}S \leq 3{:}1$, primitive only)

L/S pattern	Durations	Cycle	Ratio
Two elements (1 Long, 1 Short)			
LS	2+1	3	2
LS	3+1	4	3
LS	3+2	5	1.5
LS	4+3	7	1.33
LS	5+2	7	2.5
LS	5+3	8	1.67
LS	5+4	9	1.25
Three elements (1 Long, 2 Short)			
LSS SLS	2+1+1 1+2+1	4	2
LSS SLS	3+1+1 1+3+1	5	3
LSS SLS	3+2+2 2+3+2	7	1.5
LSS SLS	5+2+2 2+5+2	9	2.5
Three elements (2 Long, 1 Short)			
LLS LSL	2+2+1 2+1+2	5	2
LLS LSL	3+3+1 3+1+3	7	3
LLS LSL	3+3+2 3+2+3	8	1.5
Four elements (1 Long, 3 Short)			
LSSS SLSS	2+1+1+1 1+2+1+1	5	2
LSSS SLSS	3+1+1+1 1+3+1+1	6	3
LSSS SLSS	3+2+2+2 2+3+2+2	9	1.5
Four elements (2 Long, 2 Short)			
LLSS LSLS LSSL	2+2+1+1 2+1+2+1 2+1+1+2	6	2

L/S pattern	Durations	Cycle	Ratio
SLLS	1+2+2+1		
LLSS	3+3+1+1	8	3
LSLS	3+1+3+1		
LSSL	3+1+1+3		
SLLS	1+3+3+1		
Four elements (3 Long, 1 Short)			
LLLS	2+2+2+1	7	2
LLSL	2+2+1+2		

Table 2 — transition along a rail (minimal-contrast series, $l = s+1$)

The elementary move, tabulated. Fix one shape and climb the pulse at the gentlest contrast: each block is a *trajectory to ever-higher cycle numbers*, the jump per step set by the rhythm's *density* — its cell-count n (+2 at two cells, +3 at three, +4 at four). **Error** = each internal onset minus the straight k/n grid, as % of a *unified bar*; + late, − early; the shift between two rows is the difference of their Errors, shrinking toward 0 (straight) as the cycle grows — so the further you ride a rail, the smaller the adjustment each step asks of the hands.

L/S pattern	Durations	Cycle	Error (onset dev. vs straight)
Two elements — onset vs the half-bar (50%)			
LS	3+2	5	+10.0%
	4+3	7	+7.1%
	5+4	9	+5.6%
	6+5	11	+4.5%
	7+6	13	+3.8%
Three elements — onsets vs the thirds grid (33.3%, 66.7%)			
LLS	2+2+1	5	+6.7% / +13.3%
	3+3+2	8	+4.2% / +8.3%
	4+4+3	11	+3.0% / +6.1%
	5+5+4	14	+2.4% / +4.8%
LSL (palindrome)	2+1+2	5	+6.7% / −6.7%
	3+2+3	8	+4.2% / −4.2%
	4+3+4	11	+3.0% / −3.0%
	5+4+5	14	+2.4% / −2.4%
LSS	2+1+1	4	+16.7% / +8.3%
	3+2+2	7	+9.5% / +4.8%
	4+3+3	10	+6.7% / +3.3%
	5+4+4	13	+5.1% / +2.6%
Four elements — onsets vs the quarters grid (25%, 50%, 75%)			
LLLS	2+2+2+1	7	+3.6% / +7.1% / +10.7%
	3+3+3+2	11	+2.3% / +4.5% / +6.8%
	4+4+4+3	15	+1.7% / +3.3% / +5.0%
	5+5+5+4	19	+1.3% / +2.6% / +3.9%
LLSS	2+2+1+1	6	+8.3% / +16.7% / +8.3%
	3+3+2+2	10	+5.0% / +10.0% / +5.0%
	4+4+3+3	14	+3.6% / +7.1% / +3.6%
	5+5+4+4	18	+2.8% / +5.6% / +2.8%
LSSL (palindrome)	2+1+1+2	6	+8.3% / 0.0% / −8.3%
	3+2+2+3	10	+5.0% / 0.0% / −5.0%
	4+3+3+4	14	+3.6% / 0.0% / −3.6%

L/S pattern	Durations	Cycle	Error (onset dev. vs straight)
	5+4+4+5	18	$+2.8\%$ / 0.0% / -2.8%
LSSS	2+1+1+1	5	$+15.0\%$ / $+10.0\%$ / $+5.0\%$
	3+2+2+2	9	$+8.3\%$ / $+5.6\%$ / $+2.8\%$
	4+3+3+3	13	$+5.8\%$ / $+3.8\%$ / $+1.9\%$
	5+4+4+4	17	$+4.4\%$ / $+2.9\%$ / $+1.5\%$

Table 3 — transition across the ratios (by pattern, sorted by ratio)

The move that changes rails. Every realization of one L/S shape grouped together and ordered by ratio, low to high. Read down a block: each step to the neighbouring ratio is a *natural jump between cycle numbers* — the bar-length printed beside it — the most musical way to slip from one meter into another. ($s < l$, ratio $L\!:\!S \leq 3\!:\!1$, primitive only)

L/S pattern	Durations	Cycle	Ratio
Two elements (1 Long, 1 Short)			
LS	5+4	9	1.25
	4+3	7	1.33
	3+2	5	1.5
	5+3	8	1.67
	2+1	3	2
	5+2	7	2.5
	3+1	4	3
Three elements			
LLS	3+3+2	8	1.5
	2+2+1	5	2
	3+3+1	7	3
LSL	3+2+3	8	1.5
	2+1+2	5	2
	3+1+3	7	3
LSS	3+2+2	7	1.5
	2+1+1	4	2
	5+2+2	9	2.5
	3+1+1	5	3
SLS	2+3+2	7	1.5
	1+2+1	4	2
	2+5+2	9	2.5
	1+3+1	5	3
Four elements (3 Long, 1 Short)			
LLLS	2+2+2+1	7	2
LLSL	2+2+1+2		
Four elements (2 Long, 2 Short)			
LLSS	2+2+1+1	6	2
	3+3+1+1	8	3
LSLS	2+1+2+1	6	2
	3+1+3+1	8	3
LSSL	2+1+1+2	6	2
	3+1+1+3	8	3
SLLS	1+2+2+1	6	2

L/S pattern	Durations	Cycle	Ratio
	1+3+3+1	8	3
Four elements (1 Long, 3 Short)			
`LSSS`	3+2+2+2	9	1.5
	2+1+1+1	5	2
	3+1+1+1	6	3
`SLSS`	2+3+2+2	9	1.5
	1+2+1+1	5	2
	1+3+1+1	6	3

The geometry of transition

Resting on the first page's three filters — the 3:1 cap, primitives only, and one of each retrograde pair — here is what their 37-rhythm set looks like as geometry: the rails a transition can ride, the map of shortest routes between meters, and the one crossing the lattice refuses to make.

1. The lattice: rails of constant element count. Take a shape, hold its contrast at the gentlest setting Long = Short+1, and let Short grow. The bar-length is Number $= n \cdot \text{Short} + b$, where n is the cell count and b the number of Longs, so it climbs in equal steps of n. Each element count therefore lays down a family of parallel *rails* — one arithmetic progression per value of b from 1 to $n-1$ — and a rail is precisely a route you can ride without changing the gesture:

- $n=2$: $3, 5, 7, 9, 11, 13$ (all odd bars)
- $n=3$: $4, 7, 10, 13$ & $5, 8, 11, 14$
- $n=4$: $5, 9, 13$ & $6, 10, 14$ & $7, 11, 15$
- $n=5$: $6, 11$ & $7, 12$ & $8, 13$ & $9, 14$ $n=6$: $7, 13$ & $8, 14$ & $9, 15$

The characteristics this fixes, exactly:

- A rail is one gesture at neighbouring ratios; a single step along it adds exactly n to the bar and moves the ratio to its neighbour. These steps are the rows of Table 2 — the atoms every transition is made of.
- A rail of count n joins only bar-lengths in one residue class modulo n; it can never connect two lengths closer than n apart. Bars one unit apart share *no* rail — the hard crossings.
- The figure is scale-invariant: double every value and a rail maps onto itself — the additive form of musical self-similarity.

Holding the shape and sweeping the ratio instead (2:1, 3:1, 3:2, ...) runs *across* the rails: that is the down-a-block reading of Table 3, and it is how a player changes rails mid-route.

2. The ratio-eased transition map. Call one rail-step a *transition* — shift to a neighbouring ratio of some shape and land on a bar-length n away. The *transition distance* between two meters is the fewest such steps. The map below gives, for every pair of meters 3 to 13, a shortest *route* — intermediate meters only, colour showing the number of steps. It is symmetric, so only the lower triangle is shown: read column-meter → cell → row-meter (the 7/8 cell reads $7 \to 5 \to 8$; "–" marks a direct, single-rail step).

route	3	4	5	6	7	8	9	10	11	12
4	5 7									
5	–	7								
6	5 7 10	7 10	7 10							
7	5	–	–	10						
8	5	7 5	–	11	5					
9	5	7	–	11	–	5				
10	5 7	7	7	–	–	13	7			
11	5 7	7	7	–	–	–	–	6		
12	5 7	7	7	10 7	–	5 7	7	7	7	
13	5 7	7	7	10	–	–	–	–	–	7

1 direct (shared rail) 2 3 4 steps

What the map says:

- A direct (green "–") pair shares a rail and slips into one another in one move: $5 \to 8$ is a single three-cell step (the "5 against 8" of the first page), $5 \to 9$ a four-cell step, $7 \to 9$ a two-cell step.

- Every hard crossing costs at least 2: $6 \to 7$ goes via 10, and $7 \to 8$ via 5 (7 via 5 to 8). No pair of neighbouring integers is ever a single step.
- Meter 7 is the hub: in one step it reaches $4, 5, 9, 10, 11, 12, 13$, more than any other meter here. The starved corners $3, 4, 12$ ride few rails and stay 2–4 steps from most of the field.

3. The map depends on the element count. The map above is the *union* of separate sub-lattices, one per density. Restrict to a single element count n and almost everything greys out: a rail of count n joins only meters in one residue class modulo n, so each density on its own reaches just a sliver of the field. Here are the three densities of the catalogue, drawn the same way (grey = unreachable with that density alone):

n=2 — two-cell rails: only the odd meters, 3–5–7–9–11–13.

n=2	3	4	5	6	7	8	9	10	11	12
4										
5	–									
6										
7	5		–							
8										
9	5 7		7		–					
10										
11	5 7 9		7 9		9		–			
12										
13	5 7 9 11		7 9 11		9 11		11		–	

n=3 — three-cell rails: 4–7–10–13 and 5–8–11; multiples of 3 are stranded.

n=3	3	4	5	6	7	8	9	10	11	12
4										
5										
6										
7		–								
8			–							
9										
10		7			–					
11			8			–				
12										
13		7 10			10			–		

n=4 — four-cell rails: 5–9–13, 6–10, 7–11; 8 and 12 are stranded.

n=4	3	4	5	6	7	8	9	10	11	12
4										
5										
6										
7										
8										
9			–							
10				–						
11					–					
12										
13			9				–			

Read together they show why "near" is not "easy." A four-cell move always jumps by 4 within one residue class, so 5 and 6, though adjacent, lie on *different* four-cell rails: the closest a four-cell step brings you is $5 \to 9$, and to reach 6 you must change density — board the five-cell rail 6–11, or drop to the two-cell odds. "$5 \to 6$ via 9" is exactly that: ride the four-cell rail out to 9, then switch rails ($9 \to 11 \to 6$) — three steps in all. Small element counts step finely but never leave their residue class; large ones leap further and open crossings nothing else can. The composite map of Part 2 is what you get by letting the player change density freely between steps — which is to say, your transition map is only as rich as the densities you have practised.

4. Diophantine gaps: the bars no rail reaches. A meter of bar-length Number is a whole-number pair (Short, Long) solving $\text{Number} = a \cdot \text{Short} + b \cdot \text{Long}$ with $\text{Short} < \text{Long} \leq 3 \cdot \text{Short}$, so it exists only where that segment carries a lattice point. Three consequences for what you can and cannot travel to:

- **Forbidden totals.** The 3:1 cap leaves no valid pair for some lengths: no three-cell bar of 6, no five-cell bar of 10 or 15. This is the Frobenius coin problem — with only certain cell-sizes available, some totals cannot be assembled. The smallest that ever works is $n+1$.
- **The single-pair 7.** Four-cell length 7 admits only $(\text{Short}, \text{Long}) = (1, 2)$ — the four orderings of 2+2+2+1 and nothing else, the one rigid meter in the set. Its consequences return in Part 6.
- **A staircase, with gaps.** The two-cell counts run $1, 1, 1, 1,\ 2, 2, 2, 2,\ 3, 3, 3, 3, \ldots$ — one more roughly every four lengths, since the admissible wedge of ratios has a fixed width (count $\approx$ Number/4). Steady growth, pocked by the forbidden totals: lattice-point counting of the Gauss-circle / Ehrhart kind.

5. Farey ratios: the number line across the rails. The first inside-accent of a Short-then-Long cell falls at Short/(Short+Long) of the bar. Under the 3:1 cap these positions are exactly the reduced fractions between $\frac{1}{4}$ and $\frac{1}{2}$ — the start of a *Farey sequence.* It is the ruler along which a cross-rail transition is measured.

- **The mediant.** The simplest fraction between two others is $\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}$ (add tops, add bottoms); iterating it builds the Farey sequence, the Stern–Brocot tree and continued fractions. Musically, the mediant of two cells is the simplest cell heard "between" them — the cross-rail interpolation of Part 1, made exact, and the natural way-station when a transition needs one.
- **Euclidean rhythms.** The gentlest contrast spreads the Longs as evenly as possible among the Shorts. That is the *Euclidean rhythm* — $E(2, 5)$=2+3, $E(3, 8)$=3+3+2, $E(3, 7)$=3+2+2 — the cells of these tables, and the link to maximally even scales and well-formed tunings.

6. One hypothesis, stated to be tested. Two facts line up. First, 7 is the unique four-cell meter with a single value pair (Part 4) and the hub of the transition map (Part 2). Second, by parity the odd bars are better anchored: an odd length has a true centre pulse, an even one does not, and an even bar carries one fewer two-cell rhythm than its odd neighbour. Together: 7 sits at the centre of the odd rail $5-7-9-11-13$ and is its most connected meter. The question to test at the instrument is whether 7 acts as an *attractor* — the meter the surrounding odds resolve toward — and whether even time is reached not by dividing but by crossing from that odd rail onto the three-cell $5-8-11$ rail, the `LLS` "5 against 8" family. The catalogue gives the exact cells to try it.

The strangeness, stated plainly. Read the two grids for their single sharpest fact: the hardest transition in the whole sample is $3 \to 6$, at **four** steps ($3 \to 5 \to 7 \to 10 \to 6$) — and it is the *only* four-step crossing here. That is profoundly odd, because 6 is simply *twice* 3: musically the nearest possible relative, the octave of metre. The lattice cannot see that kinship at all, and for a precise reason from the first page — doubling is the *non-primitive scaling we deliberately removed* (6 = 2×3, like 2+4 from 1+2). Having filtered out exact multiples, the geometry has no short move that merely doubles a bar; the only way from 3 to its double is the long way round, through four unrelated meters. The simplest arithmetic relation becomes the most distant rhythmic one.

In sum: two note-lengths and one 3:1 cap turn rhythm into a lattice of rails — arithmetic progressions stepped by element count, gapped where Diophantine arithmetic forbids a meter, indexed across by the Farey ratios. Every transition in this sheet is a walk through that lattice: the tables give its steps, the map gives its shortest routes, and its longest walk is the octave it refuses to take.